

Parametrizing the Neutrino sector*

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The original Standard Model has massless neutrinos, but the observation of neutrino oscillations requires that neutrinos are massive. The simple extension of adding gauge singlet fermions to the particle spectrum allows normal Yukawa mass terms for neutrinos. The seesaw mechanism then suggests an explanation for the observed smallness of the neutrino masses. After reviewing the framework of the seesaw we suggest a parametrization that directly exhibits the smallness of the mass ratios in the seesaw for an arbitrary number of singlet fermions and we present our plans to perform calculations for a process that might be studied at the LHC.

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1. Space-time and fermions

In the last century symmetries became more and more important to describe nature. Particle physics in particular experienced the need to use the symmetry group of Special Relativity from the beginning, as subatomic particles travel most of the time at velocities close to the speed of light. This in turn required that particles are described as representations of the homogeneous Lorentz group $SO(3,1)$, which is locally isomorphic to the product of two rotation groups: $SO(3,1) \sim SU(2)_L \otimes SU(2)_R$. This product representation tells, that apart from the scalar, which does not transform under Lorentz transformations, the next simple object can be a spinor, that transforms under only one of these two rotations groups. These spinors are chiral spinors or Weyl-spinors. The more usual Dirac-spinor Ψ can be written as the direct sum of two Weyl-spinors of opposite chirality:

$$\Psi = \left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right) = \Psi_L + \Psi_R . \quad (1)$$

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The two discrete symmetries, parity P and time reversal T , act on space-time and hence on any dependence of the representation of particles, too. Charge-conjugation, on the other hand, does not act on space-time, but only on the particles, i.e. on their creation and annihilation operators. But for a Lorentz covariant description this charge conjugation should be Lorentz covariant, too. Since spinor representations are usually complex, charge conjugation understood as a complex conjugation will also influence the representation in which fermion fields are defined. Therefore one has to define a Lorentz covariant conjugation (LCC)

$$\mathcal{C} \Psi \mathcal{C}^{-1} = \Psi^c := \widehat{\Psi} = \gamma^0 \mathbf{C} (\Psi)^* = -\mathbf{C} (\bar{\Psi})^\top \quad (2)$$

for spinor fields. It turns out, that the LCC acting on chiral fermions will flip the chirality of the fermion:

$$(\Psi_L)^c = \gamma^0 \mathbf{C} (P_L \Psi)^* = \gamma^0 P_L \mathbf{C} (\Psi)^* = P_R \gamma^0 \mathbf{C} (\Psi)^* = P_R \widehat{\Psi} \quad (3)$$

These chiral fermions are the building blocks of the Standard Model (SM) [1] as can be seen in [2].

A Majorana fermion Ψ_M is constrained by a reality condition in the same way as a real scalar field compared to a complex scalar field:

$$\Psi_M = \eta_M \widehat{\Psi}_M = \eta_M \gamma^0 \mathbf{C} (\Psi_M)^* \quad (4)$$

with an arbitrary phase η_M . That reduces the four degrees of freedom of the Dirac fermion to only two degrees of freedom of a Majorana spinor, like the two degrees of freedom of a Weyl spinor. But the LCC changes the chirality. Therefore a Majorana fermion cannot be a chiral fermion. Nevertheless one can define the Majorana fermion by two chiral degrees of freedom:

$$\Psi_M = \Psi_L + \eta_M \widehat{\Psi}_L = \eta_M \widehat{\Psi}_R + \Psi_R \quad (5)$$

Since a mass term connects both chiralities, a single chiral fermion cannot support a mass term. But with a Majorana fermion one can write a mass term involving only two spinorial degrees of freedom. For a didactically extended discussion of spinors see [3].

2. The Standard Model

The Standard Model (SM) [1] is a chiral quantum gauge field theory [2]. All fields are massless and obtain mass only through the Higgs mechanism [4]. The gauge symmetry $SU(3)_{\text{color}} \times SU(2)_{\text{weak}} \times U(1)_Y$ is broken by the vacuum expectation value (vev) of the Higgs field to $SU(3)_{\text{color}} \times U(1)_{\text{em}}$. This leaves the gauge bosons of the unbroken gauge symmetries, the gluons

and the photon, massless. By the choice of the Higgs coupling to the chiral fermions, also the quarks and the charged leptons obtain a mass proportional to their coupling to the Higgs field. This coupling couples different chiral fermions and produces pairs of equal mass fermionic states, which group together to form the usual Dirac fermions. Since there is no Higgs coupling between the lepton doublets and a right chiral $SU(2)_{\text{weak}}$ singlet, the neutrinos remain massless in the "original" SM [1].

The SM exhibits an additional global symmetry $U(1)_L \times U(1)_R \sim U(1)_V \times U(1)_A$. The vector combination $U(1)_V$ enforces fermion number conservation, but the axial vector current $U(1)_A$ is anomalously not conserved by QCD quantum effects [5]. Since these quantum effects are similar to the spontaneous symmetry breaking, one would expect a Goldstone like degree of freedom, an axion. Up to now no axion has been found [6].

2.1. Adding gauge singlet fermions

One of the major new insights in the last decades is the experimental observation of neutrino oscillations [7]. But the massless neutrinos cannot oscillate. A very simple extension to the SM is the addition of gauge singlet fermions N . With these one can write down a mass term for neutrinos

$$\mathcal{L}_{\text{Yuk},\nu} = -\tilde{\phi}^\dagger \bar{N} Y_\nu L_L + h.c. , \quad (6)$$

in a similar way as for the quarks and the charged leptons

$$\mathcal{L}_{\text{Yuk}} = -\phi^\dagger \bar{\ell}_R Y_e L_L - \tilde{\phi}^\dagger \bar{u}_R Y_u Q_L - \phi^\dagger \bar{d}_R Y_d Q_L + h.c. , \quad (7)$$

where ϕ is the SM Higgs doublet, $\tilde{\phi} = i\tau_2 \phi^*$; ℓ_R , u_R , and d_R are the right handed leptons, up-type and down-type quarks, L_L and Q_L are the left handed lepton and quark doublets, and Y_k are the respective Yukawa matrices. Apart from generating the mass for neutrinos, which allows for oscillation, this addition does not affect other predictions of the overly successful SM¹. Specifically, it preserves the global chiral $U(1)_L \times U(1)_R$ symmetry.

But it leaves open the question, why the neutrino masses are that much smaller than all the other masses of the SM: after all, the masses of all particles in the SM are generated from the single vacuum expectation value v of the SM Higgs doublet.

2.2. Adding a Majorana mass term for the gauge singlet fermions

Since N is a gauge singlet field and hence electrically neutral, one can require a Majorana condition for N and define it with its chiral component:

$$N = N_L + \eta_N \widehat{N}_L = \eta_N \widehat{N}_R + N_R . \quad (8)$$

¹ For the SM being overly successful compare the talk by Leszek Roszkowski: "SUSY in the light of LHC and dark matter."

N being a Majorana fermion, one can also add a Majorana mass term

$$\mathcal{L}_M = -\frac{1}{2}N_R^\top \mathbf{C}^{-1} M_R N_R + h.c. , \quad (9)$$

which breaks the global chiral symmetry explicitly: $U(1)_L \times U(1)_R \rightarrow U(1)_V$. Therefore one no longer would expect an axion.

This change motivates us to name the model now differently: ν SM.

3. The seesaw mechanism in the ν SM

The n_R gauge singlets N have the same conserved quantum numbers as the three SM neutrinos. Describing the mixing of the neutral fermionic fields produces a $(3 + n_R) \times (3 + n_R)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} , \quad (10)$$

where M_R is the Majorana mass term, eq.(9), $M_L = 0$ at tree level, and $M_D = vY_\nu$ is the Dirac mass term from the Higgs coupling between the lepton doublet and the gauge singlets. In contrast to the usual Yukawa matrices for quarks and charged leptons, this Dirac mass term does not need to be represented with a quadratic matrix.

The most convenient diagonalization of the mass matrix M_ν , eq.(10), for arbitrary n_R is the Grimus-Lavoura ansatz [8]

$$W^\top M_\nu W = W^\top \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} W = \begin{pmatrix} M_\ell & 0 \\ 0 & M_h \end{pmatrix} \quad (11)$$

with a unitary

$$W = \begin{pmatrix} \sqrt{1 - BB^\dagger} & B \\ -B^\dagger & \sqrt{1 - B^\dagger B} \end{pmatrix} , \quad (12)$$

where B is a general complex $3 \times n_R$ matrix. With the assumption $M_R \gg M_D \gg M_L$ one can expand eq.(11) into a perturbation series and solve the series recursively for the masses M_ℓ and M_h and the mixing matrix W , which is completely determined by B . Seesaw [9] is the name for the resulting relations $M_h \approx M_R$ and $M_\ell \approx M_D^\top M_R^{-1} M_D$.

Decomposing B by a singular value decomposition

$$B = U S V^\dagger \quad (13)$$

allows us to quantify the parameters of the perturbation expansion in terms of the singular values S_j . The lowest order of eq.(11),

$$S V^\top M_h V S^\top = U^\top (M_L - M_\ell) U , \quad (14)$$

exhibits the ratio of scales in the seesaw

$$S_j^2 = \frac{[U^\top (M_L - M_\ell) U]_{jj}}{[V^\top M_h V]_{jj}} \sim \frac{\mathcal{O}(M_\ell)}{\mathcal{O}(M_h)} \sim \frac{10^{-9} \text{ GeV}}{10^{11} \text{ GeV}} \sim 10^{-20} , \quad (15)$$

expressed by the singular values S_j .

When comparing M_ℓ to the oscillation data we see two measured differences of squared mass values Δm_i^2 for the neutrinos. From that we can conclude, that at least two values of M_ℓ have to be non zero. At tree level we see, that the number of singular values $S_j > 0$ gives us the number of non zero mass values in M_ℓ . This excludes the possibility of having only a single gauge singlet giving masses to the neutrinos at tree level, i.e. $n_R = 1$. When loop corrections are included, a symmetric M_L will be generated [10], which allows more non zero mass values in M_ℓ , although the matrix $(M_L - M_\ell)$ still has only rank one and hence only a single non zero singular value.

For two added gauge singlets we can expect two non zero mass values in M_ℓ already at tree level. One neutrino would be expected to be massless. The matrices U and V , still connected by eq.(14), allow the parametrization of the neutrino sector together with the input of the two masses in M_R and the two measured differences of squared mass values. Loop corrections allow to have three non vanishing light neutrino masses.

In the usually assumed case of $n_R = 3$ one can have three non zero singular values, giving three non zero mass values in M_ℓ already at tree level. Taking Δm_i^2 and M_h as input parameters we still have to choose V and U , restricted by eq.(14), in order to define our model parameters. A more convenient parametrization, that only works in the case $n_R = 3$, is the Casas-Ibarra parametrization [11], used in [12], that solves the leading order seesaw equation

$$M_\ell = -M_D^\top M_h^{-1} M_D \quad (16)$$

by the ansatz

$$M_D = i M_h^{1/2} \cdot O \cdot M_\ell^{1/2} \quad (17)$$

with an arbitrary (complex) orthogonal matrix O . This parametrization is implicitly connected to ours by

$$i M_h^{1/2} \cdot O \cdot M_\ell^{1/2} = M_D = M_h B^\dagger = M_h V S U^\dagger . \quad (18)$$

4. Neutral fermions in the ν SM

Since only mass eigenstates describe the physical particles, we have to diagonalize the mass matrices of all fields in the ν SM. The chiral fields of quarks and charged leptons have mass matrices, that only connect the left chiral with the right chiral degrees of freedom. This pairing gives a

symmetric mass matrix with pairs of equal mass values for the left chiral and the right chiral mass eigenstates. Therefore one can describe the charged leptons and the quarks by massive Dirac spinors with four degrees of freedom each.

The n_R gauge singlets together with the three neutral leptons form also a symmetric mass matrix, but due to the Majorana mass term for the gauge singlets, the mass eigenvalues do not need to form pairs. Therefore one gets $(3 + n_R)$ massive fermions with only two degrees of freedom each, which can be understood as Majorana fermions. There will be three light mass values in M_ℓ and the n_R heavy mass values in M_h .

The oscillation relevant mixing, described by the neutrino flavor mixing matrix, or *Pontecorvo-Maki-Nakagawa-Sakata* matrix U_{PMNS} [13], comes from the coupling of the charged leptonic current

$$W_\mu^- \bar{\ell}_L \gamma^\mu P_L \nu_L + W_\mu^+ \bar{\nu}_L \gamma^\mu P_L \ell_L , \quad (19)$$

$\ell_L(\nu_L)$ being the charged (neutral) part of the lepton doublet L_L . In terms of mass eigenstates, the neutral lepton state ν_L is not made up of only the three neutrinos, but has also a tiny admixture of the singlet fermions N . Inverting the mixing of the neutral mass eigenstates

$$\chi = \begin{pmatrix} \chi_{\text{light}} \\ \chi_{\text{heavy}} \end{pmatrix} = W^\dagger \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} \quad (20)$$

with the parts of eq.(12) gives

$$\nu_L = \sqrt{1 - BB^\dagger} P_L \chi_{\text{light}} + B P_L \chi_{\text{heavy}} \approx P_L \chi_{\text{light}} , \quad (21)$$

which states, that in the basis of the neutral mass eigenstates the PMNS matrix is given by the diagonalization matrix of the charged leptons

$$vY_e = U_{eR} \cdot \text{diag}(m_e) \cdot U_{eL}^\dagger = U_{eR} \cdot \text{diag}(m_e) \cdot U_{\text{PMNS}}^\dagger . \quad (22)$$

So U_{PMNS} has its origin, like the CKM matrix, in the non-alignment of the charged lepton and neutrino mass matrices.

5. Outlook

When comparing precision measurements to the predictions of the SM, one has to include loop corrections, as exemplified in [2]. The conceptually simplest renormalization scheme is the on-shell prescription, where all external particles are physically measured. These measurements define the scale, where the counter terms for the quantum corrections can be calculated. For confined quarks this prescription cannot be used, since they cannot be

observed as free particles, but for the neutrinos the on-shell prescription should work very well.

The large difference in scales encountered in the neutrino sector suggests, that the Born approximation, i.e. using only tree level processes, should be sufficient for the calculations. But as shown in [10], though $M_L = 0$ at tree level, δM_L will receive contributions by loops with a neutral fermion and a Higgs- or Z -boson; these were calculated in a general framework by Grimus and Lavoura [14]. Since these contributions can lift the zero mass degeneracy even in the case of $n_R = 1$ they change the picture obtained in the Born approximation completely. The Majorana mass term generated in this way is of the size $(vY_e)^2/M_h$, which is the same size as the seesaw generated M_ℓ , and it can be included as an effective mass term in M_ν . The diagonalization with the Grimus-Lavoura ansatz is not changed by these quantum corrections.

We want to generalize the analysis of [12] to include the cases for $n_R = 1$ or 2. Specifically we want to look at the process

$$W^\pm \rightarrow \tau^\pm + \nu \rightarrow h_1^\pm + h_2^\mp + h_3^\pm + \nu + \nu \quad (23)$$

and study the τ polarization coming from the decay of a W at the LHC.

REFERENCES

- [1] S. L. Glashow, Nucl. Phys. **22** (1961) 579. S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264. A. Salam in *Elementary Particle Physics: Relativistic Groups and Analyticity. Eighth Nobel Symposium*, ed. N. Svartholm (Stockholm, Almqvist and Wiksell, 1968), pp. 367.
- [2] F. Jegerlehner,
Renormalizing the Standard Model
In the Proceedings of Testing the Standard Model - TASI-90: Theoretical Advanced Study Inst. in Elementary Particle Physics, Boulder, Colorado, 3-29 Jun 1990, pp 476-590; PSI-PR-91-08, Apr 1991.
- [3] P. B. Pal, arXiv:1006.1718 [hep-ph].
- [4] P. W. Higgs, Phys. Rev. Lett. **13** (1964) 508-509.
F. Englert, R. Brout, Phys. Rev. Lett. **13** (1964) 321-322.
G. S. Guralnik, C. R. Hagen, T. W. B. Kibble, Phys. Rev. Lett. **13** (1964) 585-587.
- [5] Michael E. Peskin and Daniel V. Schroeder,
An Introduction to Quantum Field Theory
Reading, USA: Addison-Wesley; ISBN 0-201-50397-2 (1995)
- [6] L. Baudis, (ed.), M. Schumann, (ed.), "Axions, WIMPs and WISPs. Proceedings, 6th Patras Workshop, PATRAS 2010, Zurich, Switzerland, July 5-9, 2010," <http://axion-wimp2010.desy.de/>
and for 2011: <http://axion-wimp.desy.de/>

- [7] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81** (1998) 1562 [arXiv:hep-ex/9807003].
- [8] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].
- [9] P. Minkowski, $\mu \rightarrow e\gamma$ at a rate of one out of 10^9 muon decays?, Phys. Lett. **B 67** (1977) 421;
T. Yanagida, in *Proceedings of the workshop on unified theory and baryon number in the universe*, O. Sawata and A. Sugamoto eds., KEK report **79-18**, Tsukuba, Japan 1979;
S.L. Glashow, in *Quarks and leptons, proceedings of the advanced study institute (Cargèse, Corsica, 1979)*, J.-L. Basdevant et al. eds., Plenum, New York 1981;
M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories*, in *Supergravity*, D.Z. Freedman and F. van Nieuwenhuizen eds., North Holland, Amsterdam 1979;
R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity violation*, Phys. Rev. Lett. **44** (1980) 912.
- [10] W. Grimus, H. Neufeld, Phys. Lett. **B486** (2000) 385-390. [arXiv:hep-ph/9911465 [hep-ph]].
- [11] J. A. Casas, A. Ibarra, Nucl. Phys. **B618** (2001) 171-204. [hep-ph/0103065].
- [12] D. Aristizabal Sierra, C. E. Yaguna, JHEP **1108** (2011) 013. [arXiv:1106.3587 [hep-ph]].
- [13] B. Pontecorvo, Sov. Phys. JETP **6** (1957) 429.
Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. **28** (1962) 870-880.
- [14] W. Grimus and L. Lavoura, Phys. Lett. B **546** (2002) 86 [arXiv:hep-ph/0207229].